

Modified Kaluza-Klein Theory, Quantum Hidden Variables and 3-Dimensional Time

Xiaodong Chen*

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In this paper, the basic quantum field equations of free particle with 0-spin, 1-spin (for cases of massless and mass > 0) and $\frac{1}{2}$ spin are derived from Einstein equations under modified Kaluza-Klein metric, it shows that the equations of quantum fields can be interpreted as pure geometry properties of curved higher-dimensional time-space. One will find that if we interpret the 5th and 6th dimension as “extra” time dimension, the particle’s wave-function can be naturally interpreted as a single particle moving along geodesic path in 6-dimensional modified Kaluza-Klein time-space. As the result, the fundamental physical effect of quantum theory such as double-slit interference of single particle, statistical effect of wave-function, wave-packet collapse, spin, Bose-Einstein condensation, Pauli exclusive principle can be interpreted as “classical” behavior in new time-space. In the last part of this paper, we will coupling field equations of 0-spin, 1-spin and $\frac{1}{2}$ -spin particles with gravity equations.

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I. INTRODUCTION

Kaluza-Klein’s theory[1] showed that five-dimensional general relativity contains both Einstein’s 4-dimensional theory of gravity and Maxwell’s theory of electromagnetism[2]. Kaluza gave physicist the hope of unifying matter and geometry. Currently, people extended Kaluza-Klein’s idea to a possible “theory of everything,” 10-dimensional superstrings. One of main difficulty of the original Kaluza-Klein’s theory was: trying to make the theory to fit on our macrocosm observation that why had no fifth dimension been observed in nature? This cause the later efforts of different versions of Kaluza-Klein’s theory such as compactified, projective and noncompactified Kaluza-Klein theory.

In another totally different area, Bohm [3] and etc. tried to find quantum hidden variables to interpret quantum physics under the language of “classical” physics. There are many difficulties Quantum Hidden Variable (QHV) theory had to solve. One of main issue is: in quantum physics, even a single particle can show non-local effect, but there is no such thing in classical physics or relativity theory. For instance, to explain the interference pattern in double-slit interference experiment, one has to accept the fact that: a single photon (or electron) has to pass both slits at the same time. In general, in quantum physics, a single particle can spread out in large area – occupy many different spatial locations at the same time.

In 1999, X.Chen [4] proposed that using extra time dimension, we can explain why a single particle shows at two different locations at the same time. The paper didn’t derive any basic quantum equations. In this paper, single free-particle equation for 0-spin, 1-spin (for massless and mass > 0) and $\frac{1}{2}$ spin will be derived by

modified Kaluza-Klein equation. Thus, we shows that the equations of quantum fields can be interpreted as pure geometry properties of curved higher-dimensional time-space. If we assume that the 5th and 6th “Kaluza like” dimensions are time dimensions, then the physical effects of these extra dimensions will show basic behavior of quantum particles. Wave-function of single particle becomes geodesic path in 6-dimensional modified Kaluza-Klein time-space. In section (II), 0-spin single free particle equation (Klein-Gordon equation) with mass > 0 is derived through modified Kaluza-Klein metric. Section (III) will discuss the detail of using the two extra time dimensions to interpret basic quantum effects such as double-slit interference experiment, statistical effect of wave-function, Bose-Einstein condensation, Pauli exclusive principle. In section (IV), Maxwell’s theory of electromagnetism will be re-derived from modified Kaluza-Klein equations. The equation of 1-spin’s free single particle with mass $m_0 > 0$ will also be derived, the mass part of U(1) gauge field will be naturally included as derivative of 6th dimension. In section (V), we will obtain Dirac field equation of single particle with $\frac{1}{2}$ spin through 6-dimensional Einstein equations. In section (VI), we will coupling field equations of 0-spin, 1-spin and $\frac{1}{2}$ -spin particles with gravity equations.

II. EQUATIONS OF 0-SPIN FREE PARTICLE

The original Kaluza metric can be written as follows [2]:

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + \kappa^2 \phi^2 A_\alpha A_\beta & \kappa \phi^2 A_\alpha \\ \kappa \phi^2 A_\beta & \phi^2 \end{pmatrix}, \quad (1)$$

where the $\alpha\beta$ -part of \hat{g}_{AB} with $g_{\alpha\beta}$ (the four-dimensional metric tensor), the $\alpha 4$ -part with A_α (the electromagnetic potential), and the 44 -part with ϕ (a scalar field). The four-dimensional metric signature is taken to be $(+ - - -)$

*2073B Vestavia Park Ct, Birmingham, AL 35216, U.S.A.; Electronic address: xiaodong.chen@gmail.com

In this section, we'll only focus on obtaining field equation of 0-spin free particle, so we ignore vector field A_α , (equation with 1-spin particle will be discussed in section (IV)). Furthermore, We will add one more extra dimension than original Kaluza metric, i.e. totally 6-dimensional time-space with conditions:

$$\partial_4 g_{44} = 0, \quad \partial_5 g_{44} = -i a_5 g_{44}, \quad g_{55} = -1 \quad (2)$$

where a_5 is constant. For free particle, we ignore gravity field, then $g_{\alpha\beta} = \delta_{\alpha\beta}$, 6-dimensional time-space metric becomes:

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} & & \\ & g_{44} & \\ & & -1 \end{pmatrix} \quad (3)$$

The 6-dimensional Ricci tensor and Christoffel symbols are defined in terms of the metric exactly as in four dimensions:

$$\begin{aligned} \hat{R}_{AB} &= \partial_C \hat{\Gamma}_{AB}^C - \partial_B \hat{\Gamma}_{AC}^C + \hat{\Gamma}_{AB}^C \hat{\Gamma}_{CD}^D - \hat{\Gamma}_{AD}^C \hat{\Gamma}_{BC}^D, \\ \hat{\Gamma}_{AB}^C &= \frac{1}{2} \hat{g}^{CD} (\partial_A \hat{g}_{DB} + \partial_B \hat{g}_{DA} - \partial_D \hat{g}_{AB}) \end{aligned} \quad (4)$$

where A, B.. run over 0,1,2,3,4,5.

The 6-dimensional Einstein equations are

$$\hat{G}_{AB} = \kappa T_{AB}, \quad (5)$$

where T^{AB} is 6-dimensional energy momentum tensor, $\hat{G}_{AB} \equiv \hat{R}_{AB} - \hat{R} \hat{g}_{AB}/2$ is the Einstein tensor, \hat{R}_{AB} and $\hat{R} = \hat{g}_{AB} \hat{R}^{AB}$ are the 6-dimensional Ricci tensor and scalar respectively, and \hat{g}_{AB} is the 6-dimensional metric tensor, A, B.. run over 0,1,2,3,4,5.

Using metric (3), we get following non-zero Christoffel symbol:

$$\begin{aligned} \hat{\Gamma}_{4\alpha}^4 &= \frac{1}{2} g^{44} \partial_\alpha g_{44}, \quad \hat{\Gamma}_{\alpha 4}^4 = \hat{\Gamma}_{4\alpha}^4 \\ \hat{\Gamma}_{44}^\alpha &= -\frac{1}{2} g^{\alpha\alpha} \partial_\alpha g_{44}, \quad \hat{\Gamma}_{45}^4 = \frac{1}{2} g^{44} \partial_5 g_{44} \\ \hat{\Gamma}_{54}^4 &= \hat{\Gamma}_{45}^4, \quad \hat{\Gamma}_{44}^5 = -\frac{1}{2} g^{55} \partial_5 g_{44} \end{aligned} \quad (6)$$

Start from here, through out this paper, capital Latin indices A,B,C .. run over 0,1,2,3,5 (A,B <> 4), Greek indices $\alpha, \beta \dots$ run over 0, 1, 2, 3, and small Latin indices a, b, ... run over 1, 2, 3.

Let

$$\begin{aligned} g_{44} &= (\phi(x_0, x_1, x_2, x_3) e^{-i(a_5 x_5)})^2 \\ g^{44} &= (\phi^*(x_0, x_1, x_2, x_3) e^{i(a_5 x_5)})^2 \end{aligned} \quad (7)$$

where ϕ^* is complex conjugation of ϕ , and

$$\phi = e^{-i(a_0 x_0 - a_1 x_1 - a_2 x_2 - a_3 x_3)} \quad (8)$$

where a_α is constant.

Substitute (7) and (6) in (4):

$$\begin{aligned} R_{\alpha\beta} &= -(\phi^* \partial_\alpha \phi)(\phi^* \partial_\beta \phi) \\ R_{\alpha 5} &= R_{5\alpha} = -(\phi^* \partial_\alpha \phi)(\phi^* \partial_5 \phi) \\ R_{55} &= -(\phi^* \partial_5 \phi)(\phi^* \partial_5 \phi) \\ R_{\alpha 4} &= R_{4\alpha} = 0 \\ R_{44} &= -\phi(\partial^\alpha \partial_\alpha \phi) - \phi(\partial^5 \partial_5 \phi) \end{aligned} \quad (9)$$

and Ricci scalar becomes

$$R = g^{AB} R_{AB} = 0 \quad (10)$$

Here we used equation (7).

Using (9),(7), and let $a_5 = -\frac{m_0}{\hbar}$ where m_0 is rest mass of particle, \hbar is Planck constant, Einstein equations (5) become:

$$-(\phi^* \partial_\alpha \phi)(\phi^* \partial_\beta \phi) = \kappa T_{\alpha\beta} \quad (11)$$

$$i \frac{m_0}{\hbar} \phi^* \partial_\alpha \phi = \kappa T_{5\alpha} = \kappa T_{\alpha 5} \quad (12)$$

$$\left(\frac{m_0}{\hbar}\right)^2 = \kappa T_{55} \quad (13)$$

$$\kappa T^{4\beta} = \kappa T_{\alpha 4} = 0 \quad (14)$$

$$-\frac{1}{\hbar^2} \partial^\alpha \partial_\alpha \phi - \left(\frac{m_0}{\hbar}\right)^2 \phi = 0 \quad (15)$$

Here we let $T_{44} = 0$, i.e. no 5-dimensional energy momentum tensor. Equation (15) is Klein-Gordon Equation for free 0-spin particle. Actually, it is reasonable to let $T_{\alpha\beta} = p_\alpha p_\beta$, p_α is momentum vector of particle, α run over (0,1,2,3), the solution of equations (11)–(15) is:

$$\phi = e^{-i\sqrt{\kappa}(p^\alpha x_\alpha)} \quad (16)$$

a_α in (8) becomes $\frac{p_\alpha}{\hbar}$.

If we let $\kappa = -(\frac{1}{\hbar})^2$, equation (16) become plane wave-function of a single particle. We can see that if we describe quantum field in *pure geometry*, $\frac{i}{\hbar}$ plays the similar role as $\sqrt{8\pi G}$'s role in 4-dimensional Einstein equation, where G is gravational constant. The metric tensor becomes

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} & & \\ & e^{-\frac{2i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} & \\ & & -1 \end{pmatrix} \quad (17)$$

The interval

$$ds^2 = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 + e^{-\frac{2i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} dx_4^2 - dx_5^2 \quad (18)$$

ds^2 becomes complex function, and in this paper, we always choose $c \equiv 1$. The length can be define as

$$dl = \sqrt{|ds|^2} \quad (19)$$

where $|ds|$ is mod of ds .

III. A NEW INTERPRETATION – TIME AS 5TH AND 6TH DIMENSIONS

A. Time as quantum hidden variable

Before we discuss the meaning of metric (17), let's go back to basic quantum physics. In double-slit interference experiment of photons, it is well known that to get interference pattern, one has to assume that both slits affect each single photon, even if we make light beam weak enough to emit almost just 1 photon each time, we will still get interference fringes, but if we tried to measure which slit photon passed, the interference fringes will be destroyed. If one tries to use path to describe the movement of a single photon, he has to say that: a single photon passes two slits at the same time ! It sounds against our common physics law, that's why in current quantum physics, we have to say that there is no path for quantum particle, even in Feynman's path integral, we have to interpret that the path is imaginary path, it is not the real path of particle's movement. Similar non-localized behavior can be found in Bose-Einstein condensation and Superconductivity theory, one has to assume that the wave function of each single particles spread out the whole lattice — each single particles is everywhere in lattice, it is so called identity particles. Should we satisfy the answer from traditional quantum physics: there is no path in quantum world? Or there are some mysterious paths which we haven't understood yet? Quantum hidden variable theory tried to find the hidden variables so that we can describe the quantum effect by using classical physics language (including "path" concept). No quantum hidden variable theory gives us a satisfied answer yet.

Let's re-phrase the statement we discussed: *To use path to describe a single particle's motion, we have to say: 1 particle can occupied 2 (or more) spatial locations at the same time.* It is against our common sense – how can a particle shows up in two locations at the same time and it is still the same particle ? If you see Bob at Las Vegas at 9:00 am Jan. 1st, 2004 central time, your brother see Bob at New York at 9:00 am Jan. 1st 2004 central time, how can you tell that you and your brother are seeing the same Bob, not Bob's twin brother? The way to find out is: if you break Bob's left hand one minute later (at 9:01 am) in Las Vegas (as an extreme way to affect the physical state of the object), then if your brother see that Bob's left hand suddenly broken at 9:01 am in New York, then he is the same Bob (his physical state affected by your behavior), but if the Bob who your brother seeing is still with a perfect left hand, then he must be Bob's twin brother, not Bob himself. Suppose that your brother does see Bob's left hand suddenly broken in New York, then there must be something wrong in our time-space. First we are sure you and your brother are seeing the same Bob (broken left hand), then you and your brother are seeing Bob at different spatial locations (x_1, x_2, x_3) (Las Vegas and New York), then you and your brother are seeing Bob

at the same time t – Bob can not travel faster than light, so we have way to ensure that time is the same (ignore the gravity of the earth). How could we interpret this? Let us ask a question, is the same time t means the same time? If time is more than 1 dimension, there is another hidden dimension of time t_θ , so that Bob in New York at time $(t, t_{\theta 2})$, Bob in Las Vegas at time $(t, t_{\theta 1})$. Bob uses $t_{\theta 2} - t_{\theta 1}$ traveling from Las Vegas to New York and then travel back and forth, but we don't have any apparatus to measure t_θ , also since it is t_θ , not t , we can not use speed to measure the travel from to spatial location by t_θ . If Bob accidentally dead in Las Vegas at 9:02 am, he can not travel to New York through second time dimension t_θ , so he will disappear in New York at 9:02 am, which is called wave-package collapse in quantum physics.

To understand the above discussion better, we need understand what will happen if there are extra time dimensions. In our 1-dimensional time world, time tells us the order of events happened. We are doing things always in the time order. We can not travel to two different places at the same time because of 1 dimensional time; Time is the only parameter to describe the motion of object. If there is an extra time dimension, that means the order of a serial events happened for an object will be described by two parameters Then event A and event B which happed at the same time t in 1 dimensional time world, could actually happened at two different times in 2-dimensional time world – A and B at the same time t_1 , but different time t_2 . i.e. Serial events in 2 dimensional time, could be seen as parallel events if you only use 1 dimension clock. The new time t_2 is hidden order of even happening in our 1-dimensional time world.

Imagine if we only have 1 dimensional knowledge of the world but the actually world is 3-dimension, so we use ruler to measure space and we do not have concept of direction, then if we pick up two points on a circle with angle θ , ($0 < \theta < \frac{\pi}{2}$) and our the measurement start from center of circle, by using 1-dimensional language, we will think that the two points are the same point since the distance is the same.

Extending the above discussions about Bob to quantum world: a single particle can occupy more than one spatial locations at the time t , if we change the state of the particle in one location $X(x_1, x_2, x_3)$ at time t , its state in another location $Y(y_1, y_2, y_3)$ will be changed at the same time t . If we localized the particles location at $X(x_1, x_2, x_3)$ at time t , the particle can not be shown at location $Y(y_1, y_2, y_3)$ at time t anymore. *These are the real phenomena observed in quantum world!* It is only valid in quantum world, since wave-length $\lambda = \frac{h}{p}$, where p is momentum of particle, in macrocosm world, p is too big and λ is too small. To interpret this by using language of "classical" physics, we need two hypotheses:

- 1) There is at least one extra time dimension in our world, the new time dimension (or dimensions) acts like Kaluza's 5th dimension, it is a loop wrapped around the rest 4-dimension time-space.
- 2) A particle's motion is determined by its local curved

geometry properties of multiple time dimensions + 3 dimensional spaces; A particle moves along geodesic path in its local curved time-space.

If the first hypotheses is true, then “a particle shows in more than one locations” at the same time t is because of extra time dimensions. The particle moves from one location to another location at the same time t by extra time dimension t_2 . Later, we will prove that “the path of particle’s motion by extra time dimension” is plan-wave and it is geodesic in 6-dimensional time-space.

Should we assume the extra time dimension is a “small” loop? It is not necessary. If the particle always stays in the same location at time t in all 2nd and 3rd time dimensions t_2, t_3 , then the particle will behave the same as an object in classical physics, so even if 2nd or 3rd time dimension is big, as long as the whole loop of extra dimensional time t_2 and t_3 always stay in one location at each first dimensional time t , we will not see any effect of extra dimensional time. The curvature of local time-space is in quantum level – “small”. The reason we can not see extra time dimension in macrocosm is because: all objects’ motion in our macrocosm world are collective motion of enormous particles, the effect of the extra time dimension of each particles counterpart to each other. In addition, in Einstein theory, gravity potential is caused by curved time-space, we extend Einstein’s idea: a particle’s energy makes its local space curved. As we all know that, even in non-relativity quantum physics, a particle’s energy is always > 0 . If the above two hypotheses are correct, then a particle’s local time-space is always curved: no curved time-space, no quantum effect of particle, then particle can not exist. Based on two hypotheses, we can start interpret quantum phenomena.

B. Geodesic of 6-dimensional time-space

Now we are ready to use the results in section II. It gives us the 6-dimensional metric and equations of 0-spin particle’s local time-space. By using the results in section II, we are able to show that wave-function of particle is geodesic in 6-dimensional time-space.

The 6-dimensional geodesic equation is :

$$\frac{d^2 x^A}{d\tau^2} + \Gamma_{BC}^A \frac{dx^B}{d\tau} \frac{dx^C}{d\tau} = 0 \quad (20)$$

where τ is affine parameter. Using equation (6), above equation becomes:

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma_{44}^\alpha \frac{dx^4}{d\tau} \frac{dx^4}{d\tau} = 0 \quad (21)$$

$$\frac{d^2 x^4}{d\tau^2} + 2\Gamma_{4\alpha}^4 \frac{dx^4}{d\tau} \frac{dx^\alpha}{d\tau} = 0 \quad (22)$$

where α run over 0,1,2,3,5.

To find the solution for (20), we need find the relations between x^A and τ . let:

$$x^\alpha = \frac{-ip^\alpha}{2\hbar} \tau^2 + constant \quad (23)$$

where α runs over 0,1,2,3.

$$x^5 = \frac{-im_0}{2\hbar} \tau^2 + constant \quad (24)$$

and

$$x^4 = e^{\frac{i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} \tau + constant \quad (25)$$

the constants in (23), (24), (25) are determined by initial value of x^A . Then:

$$\begin{aligned} \frac{dx^4}{d\tau} &= e^{\frac{i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} \left(1 + \frac{i}{\hbar} (p^\alpha \frac{dx^\alpha}{d\tau} - m_0 \frac{dx^5}{d\tau}) \tau\right) \\ &= e^{\frac{i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} \left(1 + \left(\frac{i}{\hbar}\right)^2 (p_0^2 - p_a^2 - m_0^2) \tau^2\right) \end{aligned} \quad (26)$$

where a runs over 1,2,3. After use the relation: $p_0^2 = p_1^2 + p_2^2 + p_3^2 + m_0^2$, equation (26) becomes:

$$\frac{dx^4}{d\tau} = e^{\frac{i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} \quad (27)$$

Using similar algebra, we can get:

$$\frac{d^2 x^4}{d\tau^2} = 0 \quad (28)$$

It is easy to examine that, (23) (24) (25) are the solutions of (20). i.e. τ is affine parameter of geodesic and the equations above are solutions of geodesic. Rewrite (23):

$$\frac{dx^\alpha}{d\tau} = \frac{-ip^\alpha}{\hbar} \tau \quad (29)$$

Rewrite (25) and (26):

$$\tau = e^{\frac{-i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} x^4 \quad (30)$$

$$\frac{d\tau}{dx^4} = e^{\frac{-i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} \quad (31)$$

Then using (29), (31) :

$$\frac{dx^\alpha}{dx^4} = \frac{dx^\alpha}{d\tau} \frac{d\tau}{dx^4} = \frac{-ip^\alpha}{\hbar} e^{\frac{-2i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} x^4 \quad (32)$$

Similarly:

$$\frac{dx^5}{dx^4} = \frac{im_0}{\hbar} e^{\frac{-2i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} x^4 \quad (33)$$

Thus:

$$\begin{aligned} ds &= \sqrt{1 - \frac{(g_{\alpha\alpha} p^\alpha p^\alpha - m_0^2)}{\hbar^2}} e^{\frac{-4i}{\hbar} p^A x_A} x_4^2 e^{-\frac{i}{\hbar} p^A x_A} dx_4 \\ &= e^{\frac{-i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)} dx_4 \end{aligned} \quad (34)$$

where A,B runs over 0,1,2,3,5, and α runs over 0,1,2,3 and $p^5 = -m_0$, so 6-dimensional geodesic becomes plane

wave-function by dx_4 . In addition, from (23), one can see:

$$\frac{dx^\alpha}{dt} = \frac{p^\alpha}{p^0} = v \quad (35)$$

where we let speed of light $c = 1$, v is “classical” speed of particle.

Re-write equation (32) as:

$$\Delta x_\alpha = \frac{ip^\alpha}{2\hbar} e^{\frac{-2i}{\hbar} p^A x_A} (g^{44})^2 \Delta(x_4)^2 = \frac{ip^\alpha}{2\hbar} e^{\frac{2i}{\hbar} p^A x_A} \Delta(x_4)^2 \quad (36)$$

It gives us

$$\begin{aligned} |\Delta(x_4)^2| &= \left| \frac{-2i\hbar}{p^\alpha} e^{\frac{-2i}{\hbar} p^A x_A} \right| |\Delta x_\alpha| \\ &= \left| \frac{-2i\hbar}{p^\alpha} \psi^2 \right| |\Delta x_\alpha| \end{aligned} \quad (37)$$

where $\psi = e^{\frac{i}{\hbar}(p^\alpha x_\alpha - m_0 x_5)}$ is wave-function. We need this equation in next sub-section.

Finally, from (25), one can see that: in vacuum, without any particle, $p_A = 0$, then $dx_4 = d\tau$. In relativity, proper time τ_p is affine parameter, time t has relationship with τ_p : $d\tau_p = \sqrt{(1-v^2)}dt$. Time t becomes τ_p in special frame that $v = 0$. Now we see similarity between x_4 and x_0 – under certain condition, they both become affine parameter. Equation (34) tells us that x_4 alone can not keep ds at fixed value, when x_4 unchange, we still can change other x_0 and x_5 to make particle’s ds change (the other three spacial coordinate dx_1, dx_2, dx_3 should depend on time coordinates), so we have three parameters to describe particle’s motion (or “order” of events) – 3 dimensional time.

C. Quantum effects of 6-dimensional time-space

Before we go further, we need discuss the measurement of particle.

How do we measure a particle in 6-dimensional time-space? We use an apparatus to measure a particle P, first the apparatus need meet particle P, that means at least one particle A of apparatus must meet the particle P at some points of 6-dimensional time-space; Or in other words, particle A’s geodesic and particle P’s geodesic must cross each other at some 6-dimensional points. At those points, the 6-dimension coordinates of particle P equals to 6-dimensional coordinates of particle A. If at spatial location $X(x_1, x_2, x_3)$ at time t (through this paper, we always use X as 3-space dimension and t as first time dimension), the 2nd and 3rd time coordinate of particle P is (x_{4P}, x_{5P}) , and the 2nd and 3rd time coordinate of particle A is (x_{4A}, x_{5A}) , if $x_{4P} \neq x_{4A}$ or $x_{5P} \neq x_{5A}$, then even if P and A both show at location X at the same time t , they do not meet (they are different points in 6-dimensional time-space), so A and P can not “see” each other. Therefore, the chance of find P at (X, t) is not determined, it is rather a statistical result.

As we discussed before, P’s geodesic is not a straight line, if it passes (X, t) more than once through 2nd and 3rd time dimensions, then particle A has better chances to meet P. The chances of meeting P at (X, t) is depended on how many (x_{4P}, x_{5P}) of particle P “pass” (X, t) . i.e., the possibility of A meet P at (X, t) is proportional to the density of (x_{4P}, x_{5P}) pass through (X, t) . From equation (37), in a small area Δx_α , the density $|\Delta(x_4)^2|$ is proportional to $|\psi^2|$, since the possibility of finding P in Δx_α is proportional to density of x_4 , then the possibility of finding P in Δx_α is proportional to $|\psi^2|$. Now we get the same conclusion as the statistical interpretation of wave-function! We don’t need considering the density of x_5 because: we chose a special coordinate system to make metric equals (3), under this metric, the geodesic only depends on x_4 as we saw in (34).

In double slits experiment, a particles path is splitted into two paths, the particle will stay in path 1 in some of x_4 , and stay in path 2 in the same portion of x_4 , after pass the double slits, the two paths will interference each other, by equation (37), we have

$$|\Delta(x_4)^2| = \left| \frac{2\hbar}{p^\alpha} \Delta x_\alpha |\psi_{path1} + \psi_{path2}| \right|^2 \quad (38)$$

At some spatial points (on the screen), $|\psi_{path1} + \psi_{path2}|$ becomes zero, at those points, equation (38) could not be true for any non-zero $|\Delta(x_4)^2|$, it means that the density of x_4 is always zero at those points – the particle does not go to those points. That explains the minima in interference fringes.

Equation (38) also tells that at the same first dimension time t , X can be different value for different x_4 , so the particle’s position X is uncertain in 4-dimensional time-space language, but it is unique in 3+3 dimensional time-space, i.e. at each 3-dimensional time (x_0, x_4, x_5) , particle P is always in 1 spatial location $X(x_1, x_2, x_3)$. We can imagine that, if we modify metric g_{44} to the combination of different $\psi(p_i^\alpha)$, then particle can be at different momentums at the same time t .

Why we always only get 1 “dot” at interference fringes for each photon? Why 1 photon can not produce 2 “dots” on the screen? To produce a “dot” on the screen, the screen has to interact with photon, the interaction localized the photon and change photon’s local curvature of time-space of photon. Let’s say the interaction happened on 6-dimensional coordinate $A_0(x_0, x_1, x_2, x_3, x_4, x_5)$, which is spatial location $X_0(x_1, x_2, x_3)$ at 3-dimensional time (x_0, x_4, x_5) . After the interaction, the curvature of local time-space of particle P changed, particle P can not move to another screen location $X_1(x_1, x_2, x_3)$ through x_4 , the original geodesic of the particle is cut by interaction. That corresponds to wave-packet collapse in quantum physics.

There are two most important properties of 3-dimensional time:

- 1) a particle can occupy more than 1 locations at the same 1st dimensional time t .
- 2) Many particles can occupy the same location X at the same 1st dimensional

time t .

The first property gives us the non-local results of quantum physics and statistical interpretation of wavefunction. The second property will give us Bose-Einstein condensation. Considering two particles occupy location X at the same t , but their (x_4, x_5) are always different at (X, t) , then those two particles can not “see” each other, and they can not interact with each other (unless through 3rd particle). In a very small 3-D ball with many particles inside, if we can find a distribution of 2nd and 3rd time dimension of those particles such that $(x_4, x_5)_i <> (x_4, x_5)_j$ always true at any time t and at any X (i, j are indices of particle), then those particles do not interact each other. If such distribution of $(x_4, x_5)_i$ exists, we get Bose-Einstein condensation for those particles. Does such distribution exist? If at time t , projection of the 6-dimensional geodesic to 3-D ball is always a loop (U(1) symmetry) at fixed time t , we can easily put many loops to SO(3) without cross each other, so all particles do not “meet” each other, we get Bose-Einstein condensation. But if the geodesic of the particle is not a loop, for example, if the geodesic of particle has SU(2) symmetry, as we know, we can find a map from SU(2) to SO(3) which is double cover, i.e. one X only contains two different points of SU(2), suppose the two different points $(x_0, x_4, x_5)_i, (x_0, x_4, x_5)_j$ from different particle, then only 2 particles can put in a small space without meet with each other, this will explain why we have Pauli Exclusive Principle for SU(2) field, which only allows two electrons in the same “location”, one with $\frac{1}{2}$ spin, the other one with $-\frac{1}{2}$ spin. We will talk about particle with $\frac{1}{2}$ spin later.

IV. EQUATIONS OF 1-SPIN FREE PARTICLE

A. Massless 1-spin free particle

The original 5-dimensional Kaluza field equations can be written as [2] :

$$G_{\alpha\beta} = \frac{\kappa^2 \phi^2}{2} T_{\alpha\beta}^{EM} - \frac{1}{\phi} [\nabla_\alpha (\partial_\beta \phi) - g_{\alpha\beta} \square \phi] \quad ,$$

$$\nabla^\alpha F_{\alpha\beta} = -3 \frac{\partial^\alpha \phi}{\phi} F_{\alpha\beta} \quad , \quad \square \phi = \frac{\kappa^2 \phi^3}{4} F_{\alpha\beta} F^{\alpha\beta} \quad (39)$$

where $G_{\alpha\beta} \equiv R_{\alpha\beta} - R g_{\alpha\beta}/2$ is the Einstein tensor, $T_{\alpha\beta}^{EM} \equiv g_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}/4 - F_\alpha^\gamma F_{\beta\gamma}$ is the electromagnetic energy-momentum tensor, and $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$. Here the cylinder condition is being applied, which means dropping all derivatives with respect to the fifth coordinate.

For massless 1-spin free particle, if we write 6-dimensional metric as

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + A_\alpha A_\beta & A_\alpha \\ A_\beta & 1 \\ & & -1 \end{pmatrix} \quad (40)$$

and from the relation $\hat{g}_{\alpha\beta} \hat{g}^{\alpha\beta} = \delta_\beta^\alpha$, we have inverse metric:

$$(\hat{g}^{AB}) = \begin{pmatrix} g^{\alpha\beta} & -A^\alpha \\ -A^\beta & 1 + A^2 \\ & & -1 \end{pmatrix} \quad (41)$$

the $\alpha\beta$ -, $\alpha 4$ -, and 44 -components of equations (5) become:

$$G_{\alpha\beta} = \frac{1}{2} T_{\alpha\beta}^{EM} \quad , \quad \nabla^\alpha F_{\alpha\beta} = 0 \quad , \quad F_{\alpha\beta} F^{\alpha\beta} = 0 \quad (42)$$

The second of above equations is Maxwell equation. The third of equation is true for plane electromagnetic wavefunction which is the case of single free-photon. If we ignore gravity (for free photon) in first item, Equation (42) is the equations for single 1-spin massless free particle.

B. 1-spin free particle with mass > 0

For 1-spin particle with mass $m_0 > 0$, we let

$$\hat{A}_\alpha = A_\alpha e^{\frac{i}{\hbar} m_0 x_5} \quad (43)$$

where we choose $\hbar = 1$. and

$$\hat{A}_5 = 0 \quad (44)$$

where m_0 is rest mass of particle, x_5 is 6th dimension coordinate. 6-dimensional metric for 1-spin free particle become:

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + \hat{A}_\alpha \hat{A}_\beta & \hat{A}_\alpha \\ \hat{A}_\beta & 1 \\ & & -1 \end{pmatrix} \quad (45)$$

Let $\hat{F}_{AB} \equiv \partial_A \hat{A}_B - \partial_B \hat{A}_A$, and A, B runs over $0, 1, 2, 3, 5$ ($A, B <> 4$). Energy momentum tensor

$$\hat{T}_{AB} \equiv g_{AB} \hat{F}_{CD} \hat{F}^{CD}/4 - \hat{F}_A^C \hat{F}_{BD} \quad (46)$$

where A, B run over $(0, 1, 2, 3, 5, A, B <> 4)$. So the $\alpha\beta$ -, $\alpha 4$ -, and 44 -components of 6-dimensional Einstein equations (5) become:

$$\begin{aligned} G_{\alpha\beta} &= \frac{1}{2} \hat{T}_{\alpha\beta} \quad , \quad \nabla^\alpha \hat{F}_{\alpha\beta} - m_0^2 \hat{A}_\beta = 0 \\ \frac{1}{4} \hat{F}_{\alpha\beta} \hat{F}^{\alpha\beta} - \frac{1}{2} m_0^2 \hat{A}_\alpha \hat{A}^\alpha &= 0 \end{aligned} \quad (47)$$

where we choose $\hbar = 1$. This is equations for 1-spin single particle with mass > 0 . As we see above, the particle of 1-spin obtains its mass from derivative of 6th dimension (third time dimension).

V. EQUATIONS OF $\frac{1}{2}$ -SPIN FREE PARTICLE

Equations for $\frac{1}{2}$ -spin free particles are Dirac equations:

$$(i\gamma^\nu \partial_\nu - m)\psi = 0 \quad (48)$$

Here ψ is four-component complex wave function, ν runs over 0,1,2,3. γ^i , $i = 0, 1, 2, 3$ are 4×4 complex constant matrices, satisfying the relation

$$\gamma^l \gamma^k + \gamma^k \gamma^l = 2g^{kl}I, \quad k, l = 0, 1, 2, 3. \quad (49)$$

where I is the unit 4×4 matrix, and $g^{kl} = \text{diag}(1, -1, -1, -1)$ is the metric tensor.

Dirac equations are not equations of single particle, it has 4-solutions corresponding to different states of electron. The wave-function of Dirac particle is:

$$\psi = \begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \quad (50)$$

where ϕ_α ($\alpha = 0, 1, 2, 3$) is 4-components of the 1st solution Dirac equation.

According to our hypotheses that different states of particle will have different local geometry metric, so the 4-solutions should have different metrics. Let's start from first solution with spin $\frac{1}{2}$ and positive energy: For ψ , in x_3 representation, Dirac equation can be re-write as:

$$\partial_0 \phi_0 + \partial_1 \phi_3 - i\partial_2 \phi_3 + \partial_3 \phi_2 + im_0 \phi_0 = 0 \quad (51)$$

Note: Start from this section, we will always choose $\hbar = 1$.

Now for particle with half-integer spin, Let

$$\begin{aligned} \hat{K}_0 &= C g_{00} \phi_0 e^{im_0 x_5} \\ \hat{K}_1 &= C g_{11} \phi_3 e^{im_0 x_5} \\ \hat{K}_2 &= -iC g_{22} \phi_3 e^{im_0 x_5} \\ \hat{K}_3 &= C g_{33} \phi_2 e^{im_0 x_5} \\ \hat{K}_5 &= C g_{55} \phi_0 e^{im_0 x_5} \end{aligned} \quad (52)$$

where ϕ_α is the α components of equation (50), C is constant to be determined, $g_{\alpha\alpha}$ is element of usual 4-dimensional metric (1,-1,-1,-1), $g_{55} = -1$. m_0 is rest mass of the particle.

For single free particle with $\frac{1}{2}$ -spin and positive energy, we choose metric as below:

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + \hat{K}_\alpha \hat{K}_\beta & \hat{K}_\alpha & \hat{K}_\alpha \hat{K}_5 \\ \hat{K}_\beta & 1 & \hat{K}_5 \\ \hat{K}_5 \hat{K}_\beta & \hat{K}_5 & -1 + \hat{K}_5 \hat{K}_5 \end{pmatrix} \quad (53)$$

where α, β runs over 0,1,2,3, and

$$(\hat{g}^{AB}) = \begin{pmatrix} g^{\alpha\beta} & -\hat{K}^\alpha & \\ -\hat{K}^\beta & 1 + \hat{K}_A \hat{K}^A & -\hat{K}^5 \\ -\hat{K}^5 & -1 & \end{pmatrix} \quad (54)$$

Let $\hat{E}_{AB} \equiv \partial_A \hat{K}_B - \partial_B \hat{K}_A$, and A, B runs over 0,1,2,3,5 ($A, B <> 4$). Define energy momentum tensor for half spin particle:

$$\hat{T}_{AB} \equiv g_{AB} \hat{E}_{CD} \hat{E}^{CD} / 4 - \hat{E}_A^C \hat{E}_{BD} \quad (55)$$

where A,B run over (0,1,2,3,5, A, B $<> 4$), so the AB-, A4-, and 44-components (A, B $<> 4$) of 6-dimensional Einstein equations (5) become:

$$\begin{aligned} G_{AB} &= \frac{1}{2} \hat{T}_{AB}, \quad \partial_A (\partial^B \hat{K}_B) - \partial^B \partial_B \hat{K}_A = 0 \\ \frac{1}{4} \hat{E}_{AB} \hat{E}^{AB} &= 0 \end{aligned} \quad (56)$$

To derive the equation above, we used the relation below:

$$\partial_B g^{BB} (\partial_A \hat{K}_B - \partial_B \hat{K}_A) = \partial_A (\partial^B \hat{K}_B) - \partial^B \partial_B \hat{K}_A \quad (57)$$

For free particle, it is reasonable to assume that each components of \hat{K} satisfied plane-wave condition:

$$(-\partial^\alpha \partial_\alpha - m^2) \hat{K}_\alpha = 0 \quad (58)$$

where α, β runs over 0,1,2,3, or equivalently:

$$\partial^B \partial_B \hat{K}_A = 0 \quad (59)$$

where A, B run over 0,1,2,3,5. so the second equation of equations (56) become

$$\partial_A (\partial^B \hat{K}_B) = 0 \quad \text{for all } A = 0, 1, 2, 3, 5 \quad (60)$$

i.e. $\partial^B \hat{K}_B$ does not depend on x_0, x_1, x_2, x_3, x_5 , so it is reasonable to let $\partial^B \hat{K}_B$ equals zero. (Can not be a constant other than zero because \hat{K} contains a plane-wave function part of condition (59)). Together with the third equation of equations (56), we have

$$\partial^B \hat{K}_B = 0, \quad \hat{E}_{AB} \hat{E}^{AB} = 0 \quad (61)$$

are the solutions of equation (56). Now we have 3 unknown functions: ϕ_0, ϕ_2, ϕ_3 of (52), we have above two equations, plus we need choose constant C to make the first equation of (56) becomes reasonable, and for ϕ_α , they should be normalized. Substitute ϕ_α into above \hat{K}_A , the first equation of (61) becomes Dirac equation (51), and the solutions of (61) are:

$$\begin{aligned} \phi_0 &= \sqrt{\frac{m_0 + p_0}{2m_0}}, \quad \phi_1 = 0, \\ \phi_2 &= \sqrt{\frac{m_0 + p_0}{2m_0}} \frac{p_3}{m_0 + p_0}, \\ \phi_3 &= \sqrt{\frac{m_0 + p_0}{2m_0}} \frac{p_1 + ip_2}{m_0 + p_0} \end{aligned} \quad (62)$$

$$(63)$$

The above ϕ_α is the solution of equations (56) and it is also first solution of Dirac equation (51). We choose the constant C in equations (52) as

$$C = \frac{\sqrt{(m_0 + p_0)2m_0}}{p_3} \quad (64)$$

Substitute (64), (63) into (55), then:

$$\hat{T}_{ab} = p_a p_b e^{\frac{-2i}{\hbar} p^c x_c} \quad (65)$$

where a, b, c runs over 0,1,2,3,5 and $p_5 = m_0$. The result is just we expected: the Energy-Momentum tensor becomes the product of two 5-dimensional momentum vector (p_α, m_0) times the square of plane wave function. Note: the above solutions is derived under x_3 representation of Pauli matrix, it means choose a special coordinate for our 6-dimensional metric. If we choose different representation, the relationship between \hat{K}_A and ϕ_A in (52) will be different, but equations (56), (61) does not dependent on the choice of coordinate, so we'll still get solutions of Dirac equation under new representation.

If we let

$$\begin{aligned} \hat{K}_0 &= C g_{00} \phi_1 e^{im_0 x_5} \\ \hat{K}_1 &= C g_{11} \phi_2 e^{im_0 x_5} \\ \hat{K}_2 &= i C g_{22} \phi_2 e^{im_0 x_5} \\ \hat{K}_3 &= -C g_{33} \phi_3 e^{im_0 x_5} \\ \hat{K}_5 &= C g_{55} \phi_1 e^{im_0 x_5} \end{aligned} \quad (66)$$

We can get local time-space metric of single free particle with spin $-\frac{1}{2}$, and the corresponding equation will derive the second solution of Dirac equation. Similarly, we can get the local metric of single free particle with negative energy and spin $\frac{1}{2}$:

$$\begin{aligned} \hat{K}_0 &= -C g_{00} \phi_3 e^{im_0 x_5} \\ \hat{K}_1 &= -C g_{11} \phi_1 e^{im_0 x_5} \\ \hat{K}_2 &= i C g_{22} \phi_1 e^{im_0 x_5} \\ \hat{K}_3 &= -C g_{33} \phi_0 e^{im_0 x_5} \\ \hat{K}_5 &= C g_{55} \phi_3 e^{im_0 x_5} \end{aligned} \quad (67)$$

and the third solution of Dirac equation. And

$$\begin{aligned} \hat{K}_0 &= -C g_{00} \phi_4 e^{im_0 x_5} \\ \hat{K}_1 &= -C g_{11} \phi_0 e^{im_0 x_5} \\ \hat{K}_2 &= -i C g_{22} \phi_0 e^{im_0 x_5} \\ \hat{K}_3 &= C g_{33} \phi_1 e^{im_0 x_5} \\ \hat{K}_5 &= C g_{55} \phi_4 e^{im_0 x_5} \end{aligned} \quad (68)$$

for single free particle with negative energy and spin $-\frac{1}{2}$, and the 4th solution of Dirac equation.

As we see above, we didn't get all four Dirac solutions in one metric, instead, we get each solutions of Dirac equation under each different metric, and each different metric corresponding to different state of particle. It is

reasonable because: in this paper, the basic idea is that local metric of time-space determined the state of particle, for a free single particle with one state (not combination of states), its local metric of time-space should only be one of solutions of Dirac equation. Thus, we can say that we obtained the single particle equations for the particle with $\frac{1}{2}$ -spin in this section. Note: the metric we got for $\frac{1}{2}$ -spin particle is similar to the metric for 1-spin particle except that, $\frac{1}{2}$ -spin has nonzero \hat{K}_5 components. It causes the non-diagonal elements between 5th dimension and 6th dimension. It indicates that for integer spin particle, the 5th dimension loops around 4-dimensional time-space, but for half integer spin particle, the 5th dimension loops around all other 5-dimensional time-space. The metric of 1-spin particle has symmetry of 4-dimensional time-space, but the metric of $\frac{1}{2}$ has symmetry of all 5-dimensions.

The similarity between the metric of single electron (53) and the metric of a single photon (40) makes us easier to combine electron and photon into the same metric. If we use Klein's idea [5]: the derivatives of its fifth coordinate not equal 0. We can interpret the non-zero derivatives of fifth coordinate as: The coupling between electron and photon changed local time-space metric.

Let

$$\hat{K}_A \rightarrow \hat{K}_A e^{-i\gamma x_4} \quad (69)$$

where γ is a very small constant, and there is no \hat{K}_4 . Then

$$\partial_4 K_A = -i\gamma K_A \quad (70)$$

The metric \hat{g}_{AB} of the coupling of electron-photon is:

$$\begin{pmatrix} g_{\alpha\beta} + (A_\alpha A_\beta + \hat{K}_\alpha \hat{K}_\beta) & A_\alpha + \hat{K}_\alpha & \hat{K}_\alpha \hat{K}_5 \\ A_\beta + \hat{K}_\beta & 1 & \hat{K}_5 \\ \hat{K}_5 \hat{K}_\beta & \hat{K}_5 & -1 + \hat{K}_5 \hat{K}_5 \end{pmatrix} \quad (71)$$

where α, β runs over 0,1,2,3. A_α is components of vector field of photon. \hat{K}_α satisfied one of equations (52), (66), (67), (68). From above metric, one can obtain the coupling item: $-ieA_\nu \hat{K}_\nu$ and $\partial_\alpha \rightarrow \partial_\alpha - ieA_\alpha$. The metric of interaction between electron and photon will be discussed in future.

VI. COUPLING OF QUANTUM FIELD EQUATIONS AND GRAVITY

Kaluza's original purpose is to unify gravity and Maxwell equation. Since we already have above quantum field equations and they are all derived under 6-dimensional Einstein equations, if we assume that the gravity does not change the curvature of 5th and 6th dimension, it is straight forward to combine gravity and quantum field equations for particle with 0-spin, 1-spin and $\frac{1}{2}$ -spin.

For 0-spin particle, the metric is the same

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} & & \\ & \phi^2 & \\ & & -1 \end{pmatrix}, \quad (72)$$

where $g_{\alpha\beta}$ is 4-dimensional metric of original Einstein equation. The 5-dimension and 6-dimension are diagonal, so we can separate the contribution of gravity part and 5th and 6th dimension part. Field equations becomes:

$$G_{\alpha\beta}^E + G_{\alpha\beta}^Q = 8\pi G T_{\alpha\beta}^E + T_{\alpha\beta}^Q \quad (73)$$

$$G_{5\alpha}^Q = T_{5\alpha}^Q = -(\phi^* \partial_A \phi)(\phi^* \partial_B \phi) \quad (74)$$

$$\square \phi - m_0^2 \phi = 0 \quad (75)$$

$$G_{55}^Q = T_{55}^Q = m_0^2 \quad (76)$$

where G is gravitational constant. $T_{\alpha\beta}^E$ is original Einstein energy momentum tensor, T_{AB}^Q is quantum energy momentum tensor defined in section II: $T_{AB}^Q = p_{AB}$. $G_{\alpha\beta}^E$ is original Einstein tensor, G_{AB}^Q is quantum part of

Einstein tensor which is contributions from 5th and 6th dimension:

$$G_{AB}^Q \equiv R_{AB}^Q - R^Q \hat{g}_{AB}/2 \quad (77)$$

and

$$R_{AB}^Q = -(\phi^* \partial_A \phi)(\phi^* \partial_B \phi) \quad (78)$$

As we see that Klein-Golden equation becomes equation (76).

For 1-spin particle with mass > 0 , the metric becomes

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + \kappa^2 A_\alpha A_\beta & \kappa A_\alpha & \\ \kappa A_\beta & 1 & \\ & & -1 \end{pmatrix} \quad (79)$$

where $\kappa = 4\sqrt{\pi G}$. Field equations (47) should stay the same. If $m_0 = 0$, the metric above becomes original Kaluza metric with scalar field $\phi = 1$.

For $\frac{1}{2}$ -spin particle, the metric becomes:

$$(\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + \kappa^2 \hat{K}_\alpha \hat{K}_\beta & \kappa \hat{K}_\alpha & \kappa^2 \hat{K}_\alpha \hat{K}_5 \\ \kappa \hat{K}_\beta & 1 & \kappa \hat{K}_5 \\ \kappa^2 \hat{K}_5 \hat{K}_\beta & \kappa \hat{K}_5 & -1 + \kappa^2 \hat{K}_5 \hat{K}_5 \end{pmatrix} \quad (80)$$

where $\kappa = 4\sqrt{\pi G}$. With gravity, we do not have simple relation as equation (57), since the derivative of $g^{\alpha\alpha}$ could be non-zero, we can not get free particle Dirac equation. Instead, the equations below should still valid:

$$G_{AB} = \frac{1}{2} \hat{T}_{AB}, \quad \nabla^A \hat{E}_{AB} = 0$$

$$\frac{1}{4} \hat{E}_{AB} \hat{E}^{AB} = 0 \quad (81)$$

where A, B run over 0,1,2,3,5, the definition of \hat{T}_{AB} , \hat{K}_A , \hat{E}_{AB} is defined the same as in section V.

VII. DISCUSSIONS AND CONCLUSIONS

In the most part of this paper, we derived quantum field equations of 0-spin particle, massless 1-spin particle, 1-spin particle with mass > 0 and $\frac{1}{2}$ -spin particles by using 6-dimensional metrics. The equations are derived naturally as pure geometry properties of 6-dimensional time-space. Mass is included as derivative of 6th dimension. Metric of $\frac{1}{2}$ -spin particle has the same format as metric of 1-spin except that the 6th component of vector field is not zero. Particle's wave-function becomes geodesic of 6-dimensional time-space. As the conclusion,

the field equations of those particles become *pure geometry*, that makes us easier to unify gravity and quantum fields.

All of those can stay the same without interpreting the 5th and 6th dimensions are "Time" dimension. Why we need call them extra time dimensions? To answer this question, we face with a much bigger question, what is time? It is too big question to answer here. Maybe we never get the answer. But we can show some critical properties of time. First, Time makes us know the order of events happening. We know what happened before and what happened after. In the world 1-dimensional time, we know one person can only do one thing at a time (depends on how do you define one thing). But in quantum world, as we discussed in section III, quantum non-local effect makes a particle can show in different locations at the same time, the distance of those locations can be quite large as it is shown in Bell's inequality experiment. That makes us to question that if there is more than 1 dimensional time in the world? Second, when we are talking about time in common life, we usually talk about "proper time" τ , actually time plays "affine parameter" in our time-space. In relativity, time is no longer as "affine parameter", time is "affine parameter" of geodesic only when we choose a special reference frame. As we see in section III, if we choose 5th dimension as

parameter, the particle's geodesic path is wave-function, it also naturally shows that the possibility to meet the particle is proportion to square of wave-function. If no particle exists – in vacuum, 5th dimension becomes affine parameter. That makes us believe that 5th dimension is time. Third, the extra dimensional time makes us re-define the meaning of “meet” of two particles. In the new definition that two particle can meet each other if and only if all 6 coordinates have the same value for both particle, i.e. their 6-dimensional geodesics have at least one point crossing each other. That makes us to obtain the results of Bose-Einstein condensation. Actually the two properties of extra time dimension :

1) a particle can occupy more than 1 locations at the same 1st dimensional time t . 2) Many particles can occupy the same location X at the same 1st dimensional time t .

They makes us to understand most of basic quantum phenomena. In addition, if we interpret 5th dimension as space, we have to face the same problem as Kaluza: to make 5th dimension small. As we discussed in section III, we do not need make this assumption as long as that in macrocosm world, the metric of 6-dimensional time “localized” all 3 time dimensions — the collective effects of enormous particles inside each objects in macrocosm world make all 3 time dimensions “moves” in the same behavior.

But why the 6th dimension is also time dimension. First, it is the consideration of symmetry. Since we have 3-dimensional space, and we also need at least 2 dimensional time, and we need totally 6-dimensional time-space to derive all those equations, why not the 6th dimension is time, which makes world 3D-Time + 3D-Space ? Second, as one notices that the metric is complex in this paper. That makes the definition of interval ds becomes complex. The geodesic is complex function too and only related to 5th dimension. Actually the metric in original Kaluza theory could be complex too since A_α is plane electromagnetic wave. But how do we understand complex interval? How do we understand complex geodesic ? To derive all the results in this paper based on select a

special coordinates. Can we choose different coordinate to make metric only containing real parts? We know that a complex function can be described as a real function with two components. Is it possible that the reason we have complex metric is just because of trying to derive current quantum equations; it is possible that in future, we can make the metric which only contains real part by including second and third time dimensions in geodesic?

Now let's try to understand spin. As we see in 6-dimensional time-space metric of 0-spin particle, 5th dimension is diagonal. There is no components between 4-dimension and other dimensions. In time-space metric of 1-spin particle, 5th dimension wrapped around 4 dimensional time-space, no components between 5th dimension and 6th dimension. In time-space metric of $\frac{1}{2}$ -spin particle, the time-space geometry become more complicate, 5th dimension wrapped around all other 5 dimensional time-space. In our knowledge of 4-dimensional world, if an objects moves around some space dimensions, it is a rotation movement. It is reasonable to interpret that the spin is the particle rotating the other dimensions through 5th dimension.

As a conclusion, this paper shows us that we can describe quantum particle fields by using pure geometry methods. All we needs are proper metrics and 6-dimensional Einstein field equations. The two hypotheses we used plus modified Kaluza metric are very good candidates to interpret quantum phenomena. The methods are simple in both logical and mathematical. We also demonstrate the potentials to unify gravity and other quantum fields by using just 6-dimensional time-space, since all those fields can be derived by Einstein equation. Finally, using pure geometry —Einstein equation to describe quantum particles is different from current gauge field theory, in section V, we do not use all Dirac solutions (the 4-solutions), instead each time we only need one of Dirac solutions. In other words, the methods we are using is focusing on interactions between single particles, it is particle theory rather than fields theory. We will discuss interactions between different particles in future.

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